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Addressing Fuzzy Linear Programming Problems Through Ranking Functions

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Abstract: In contemporary decision-making, reliance on information is paramount. Yet, when much of it is fraught with uncertainty, decision-making becomes challenging. To tackle this uncertainty, various methods, including the use of fuzzy numbers, have been employed. This paper specifically delves into addressing linear programming problems, characterized by fuzzy coefficients in the objective function, fuzzy values in the right-hand side, and fuzzy coefficients of constraints. The proposed approach involves employing linear ranking functions such as Maleki, Campos, Yager's F1 and Yager linear ranking functions to address these fuzzy linear programming problems and attain optimal solutions. Furthermore, the paper elucidates the resolution steps with a presentation of numerical examples. In this study, a comprehensive methodology is presented for effectively addressing a wide range of linear programming problems.

1. Introduction

Decision-making is the most important and inevitable aspect of the application of mathematical methods in various fields of human activity. In real-world situations, decisions are fuzzy, at least partly [1]. Optimization is a kind of decision-making in which decisions have to be taken to optimize one or more objectives under some prescribed set of circumstances. Fuzzy optimization is a method for dealing with the ambiguity and vagueness in uncertain parameters, represented by fuzzy elements, of which membership to a specific set is imprecise. One of the key benefits of fuzzy optimization is that it enables us to handle a wide range of uncertainties in the problem structure, such as the variability in the decision maker's aspiration level regarding the objectives, the variability in the range of coefficients of the objective function(s) and constraints, and the uncertainty in the satisfaction level of constraints, in contrast to the robust optimization approach, which captures the uncertainty in only parameters [2]. The Fuzzy Linear Programming (FLP) problem is the classical Linear Programming (LP)problem to find the (maximum & minimum) values of linear function under constraints represented by linear inequalities or equations [3].

This paper concentrates on addressing FLP problems featuring inequality constraints, where the coefficients of the objective function c_j, the right-hand side b_i,as well as coefficients of constraints a_{ij}, involve fuzzy numbers. The proposed method relies on the utilization of Maleki, Campos, Yager's F1, and Yager linear RFs, offering a straightforward and practical approach to deal with the intricacies arising from fuzzy parameters. The simplicity and applicability of these methods make them advantageous compared with existing techniques for handling FLP problems encountered in practical contexts. Many practical problems involving uncertainty can be effectively modeled as mathematical problems, demonstrating the broad applicability of the proposed methods. The paper supports its methodology by presenting numerical examples, where obtained results are thoroughly analyzed and discussed, showcasing the effectiveness and utility of the proposed approach in real-world problem-solving situations.

2. Literature Review

Multiple studies have been presented about FLP. A lot of application problems can be modeled as mathematical problems that may be formulated with uncertainty. The concept of FLP was first proposed by Zimmerann. Therefore, he is recognized as the pioneer of FLP, a concept that has seen subsequent developments in the field [3, 4]. Notable advancements include the application of LP with fuzzy parameters in industrial production planning, as investigated by Vansant *al.* [5].

Maleki [6] introduces the incorporation of fuzzy variables into LP problems and proposes a novel solving approach employing ranking functions (RFs). Pandian and Jayalakskmi [7] present a technique tailored for solving integer LP problems with fuzzy variables. Expanding on previous information, Pandian and Jayalakshmi introduce a specialized method designed to tackle integer LP problems that involve fuzzy variables. This technique represents a significant advancement in addressing optimization challenges where decision variables are characterized by fuzzy values. This may involve incorporating fuzzy logic principles to model imprecise or uncertain information, thereby enabling more realistic and robust optimization solutions.

Singh [8] introduces an innovative method for addressing fully FLP problems, employing RFS. Dheyab [9] extends the scope to fuzzy fractional LP problems, also utilizing RFs. Hashem [10] introduces a distinctive method by representing decision-makers using nonsymmetrical trapezoidal fuzzy numbers (TrFNs) and solving problems through RFs. These diverse contributions collectively enhance the methodology and applicability of FLP across various problem types and domains.

3. Materials and Methods

We present a summary of crucial concepts and principles originating from fuzzy set theory. This serves as a foundation for the subsequent discussions, offering a clear understanding of the theoretical framework involved, including:

3.1. Membership Function

This introduces the defining function, denoted as μ_A, assigning 0 or 1 to each element in the crisp set A⊆X. This function extends to ($\mu_{\tilde{A}}$) with values ranging from 0 to 1, denoting the membership grade of each $x \in X$. It is commonly known as the membership function ($\mu_{\tilde{A}}$), which defines a fuzzy set$\tilde{A} = \{(X, \mu_{\tilde{A}});\ x \in X\}$, with$\mu_{\tilde{A}}$ defining the membership grade for each $x \in X$. For every x ∈ X, the value assigned by$\mu_{\tilde{A}}$is termed the membership grade of x in $\tilde{A}$[11, 12].

3.1.1.Notation

When X is the set consists of a limited or countable number of elements $\{x_1, x_2, \dots, x_n\}$, a fuzzy of set $\tilde{A}$on X can be represented as:

$$(\tilde{A}) = \mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 + \cdots. + \mu_{\tilde{A}}(x_n)/x_n = \sum_{i=1}^{n} \mu_{\tilde{A}}\,(x_i)\,/x_i \quad (1)$$

In this expression, the notation $\mu_{\tilde{A}}\,(x_i)$ where, $i = 1,2,\dots,n$ indicates that μ_i is the membership grade of $x_i$in $\tilde{A}$, and the plus sign signifies the union operation.

When X is not finite, we write:

$$\tilde{A} = \int_{x} \mu_{\tilde{A}} \frac{(x)}{x} \tag{2}$$

3.2. Support

The support of a fuzzy set A, is the crisp set of all $x \in X$ such that $\mu_A(x) > 0$[12, 13]; see Figure 1:

$$Supp(A) = \{x \in X : \mu_A(x) > 0\} \tag{3}$$

3.3.Core

The central region of $\tilde{A}$ comprises the points x in X where $\mu_{\tilde{A}}(x)$ equals 1; see Figure 1. Symbolically, this core is denoted as [12]:

$$cor\,(A) = \{(x, \mu_{\tilde{A}}(x))/\mu_{\tilde{A}}(x) = 1\} \tag{4}$$

3.4. Normality

$\tilde{A}$ is termed Normal if $cor(A) \neq \emptyset$, meaning there exists at least one point $x \in X$ where $\mu_{\tilde{A}}(x)$ equals 1 [12].

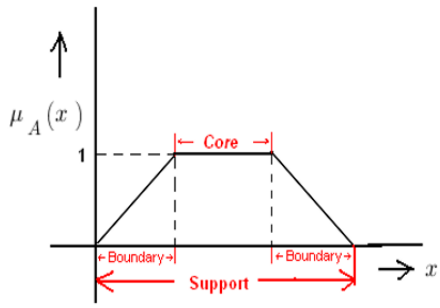

Figure 1: Core, support and boundary of a membership function representation.

3.5.λ-cut

The $\lambda - level$ (or $\lambda - cut$) set of a fuzzy set $\tilde{A}$is a crisp set denoted by $\mu_{\tilde{A}}(\lambda)$,which is defined as:

$$\tilde{A}_\lambda = \{\, x \in X \,/\, \mu_{\tilde{A}}(x) \geq \lambda \,\} \tag{5}$$

Therefore, the strong λ-cut is:

$$\tilde{A}_\lambda = \{\, x \in X \,/\, \mu_{\tilde{A}}(x) > \lambda \,\} \tag{6}$$

Where "A" is a crisp set; see Figure 2 for an illustration of the strong λ-cut[13].

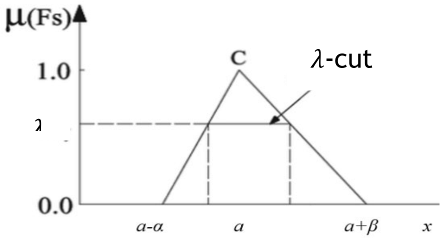

Figure 2: Illustration of the strong $\lambda - cut$ of a fuzzy set $\tilde{A}$.

3.6. Convex

A fuzzy set $\tilde{A}$ on the real line is deemed convex if $\exists\, x, y \in X$, and $\lambda \in [0,1]$. The following condition holds:

$$\mu_{\tilde{A}}(\lambda x + (1 - \lambda)y) \geq Min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\} \tag{7}$$

- A fuzzy set achieves convexity if, and only if, every $\lambda - cut$ it possesses demonstrates convexity. This observation underscores a crucial attribute of convex fuzzy sets, wherein each $\lambda - cut$, representing a horizontal slice of the set across its domain, exhibits convexity. This property is significant, as it ensures a smooth and continuous transition in the membership degrees across different elements of the set, contributing to a coherent and well-defined representation of uncertainty or imprecision.
- $\tilde{A}$ is characterized as an uncertain set on the real number line that at its core is not empty. Convexity signifying that all of its x-cuts demonstrate convexity [12, 13].

3.7.Triangular Fuzzy Number

A fuzzy set, $\tilde{A} = (a, \alpha, \beta)$, is referred to as a triangular fuzzy number (TFN) with a center a , left width α and right width β if its $\mu_{\tilde{A}}(x)$ is characterized by the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{(a-x)}{\alpha} & a - \alpha \leq x \leq a \\ 1 - \frac{(x-a)}{\beta} & a \leq x \leq a + \beta \\ 0 & o.w \end{cases} \tag{8}$$

The TFN is established upon a three-value assessment: the minimum possible value denoted as $a - \alpha$, the most probable value represented by a, and the maximum possible value indicated as $a + \beta$[7, 11].

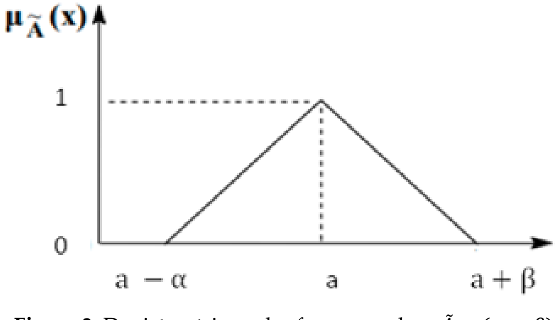

Figure 3: Depicts a triangular fuzzy number $\tilde{A} = (a, \alpha, \beta)$.

The concept of a TFN is rooted in a three-value assessment, where each value represents a distinct aspect of uncertainty. Specifically, it comprises the minimum possible value, denoted as $a - \alpha$, which signifies the lower bound; the most probable value, represented by a, which indicates the peak or center of the distribution; and the maximum possible value, indicated as $a+\beta$, serving as the upper bound. Together, these three values form the foundation of the TFN, enabling a nuanced representation of uncertainty or imprecision within a given context.

3.8.Trapezoidal Membership Function

A TrFN $\tilde{A}$ is defined by four parameters, which delineate its shape and characteristics within the fuzzy set framework:
Alower bound a^l, an upper bound a^u, and the two shape parameters α and β. Its membership function is as follows [14, 15]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1-\frac{(a^l-x)}{\alpha} & , a^l - \alpha \le x \le a^l \\ 1 & , a^l \le x \le a^u \\ 1-\frac{(x-a^u)}{\beta} & , a^u \le x \le a^u + \beta \\ 0 & , O.W \end{cases} \tag{9}$$

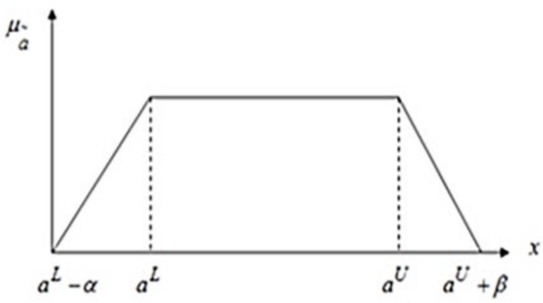

Figure 4: Shows Trapezoidal fuzzy number $\tilde{A}=(a^l,a^u,\alpha,\beta)$.

1. Drawing from the characteristics of TFNs and TrFNs, it becomes clear that a TFN is a particular instance of a TrFN, specifically where, if a^l equals a^u, the fuzzy number is identified as a TFN.
2. If α equals β, the TrFN is referred to as a symmetric.
3. A TrFN $\tilde{A}=(a^l,a^u,\alpha,\beta)$ can be characterized by its support $(a^l-\alpha,a^u+\beta)$, and its core $[a^l,a^u]$.
4. Considering $\tilde{A}=(a^l,a^u,\alpha,\beta)$ and $\tilde{B}=(b^l,b^u,\gamma,\theta)$ two TrFNs, the following arithmetic operations must be verified [15, 16, 17]:
   a. Image of $\tilde{A}$ is: $-\tilde{A}=(-a^u,-a^l,\beta,\alpha)$ (10)
   b. Addition: $\tilde{A}+\tilde{B}=(a^l+b^l,a^u+b^u,\alpha+\gamma,\beta+\theta)$ (11)
   c. Subtraction: $\tilde{A}-\tilde{B}=(a^l-b^u,\ a^u-b^l\ ,\ \alpha+\theta,\beta+\gamma)$ (12)
   d. Scalar Multiplication:

$$\text{if } x\ge 0,\ x\tilde{A}=(xa^l,\ xa^u,\ x\alpha,\ x\beta) \tag{13}$$

$$\text{If } x\le 0,\ x\tilde{A}=(xa^u,\ xa^l,-x\beta,\ -,x\alpha) \tag{14}$$

3.9. Ranking Function

The use of RFs is indeed a common approach in handling fuzzy numbers to establish a meaningful order or ranking. These functions help in comparing and ordering fuzzy numbers, which is essential in decision-making processes where uncertainty is involved. The RF is represented by F(R), where $R: F(R)\to R$, and F(R) denotes the set of fuzzy numbers defined on a real line, where a natural order prevails. Various kinds of RFs have been introduced in research, each offering distinct benefits and practical uses. These functions are designed to offer a structured approach for comparing and arranging fuzzy numbers, considering both their membership and nonmember ship values. In the

context of LP with fuzzy parameters, RFs play a crucial role in converting fuzzy constraints or objectives into crisp ones, facilitating traditional application [18, 19, 20, 21].
In the context of LP techniques, let $\tilde{A}$ and $\tilde{B}$ represent two TrFNs. The RF $F(R)$ follows the following rules:

$$\text{If } \tilde{\boldsymbol{A}} \geq \tilde{\boldsymbol{B}}, \text{ then } \boldsymbol{R}(\tilde{\boldsymbol{A}}) \geq \boldsymbol{R}(\tilde{\boldsymbol{B}}) \tag{15}$$

$$\text{If } \tilde{\boldsymbol{A}} > \tilde{\boldsymbol{B}}, \text{ then } \boldsymbol{R}(\tilde{\boldsymbol{A}}) > \boldsymbol{R}(\tilde{\boldsymbol{B}}) \tag{16}$$

$$\text{If } \tilde{\boldsymbol{A}} = \tilde{\boldsymbol{B}}, \text{ then } \boldsymbol{R}(\tilde{\boldsymbol{A}}) = \boldsymbol{R}(\tilde{\boldsymbol{B}}) \tag{17}$$

Additionally, $\tilde{A} - \tilde{B} = 0$, if and only if $R(\tilde{A}) - R(\tilde{B}) = 0$, where $\tilde{A}$ and $\tilde{B}$ are in $F(R)$.

Lemma: For any RF, the following statements hold:

1. $\tilde{A} \geq \tilde{B}$ if and only if $\tilde{A} - \tilde{B} \geq 0$ if and only if $-\tilde{B} \geq -\tilde{A}$
2. If $\tilde{A} \geq \tilde{B}$ and $\tilde{C} \geq \tilde{D}$ then $\tilde{A} + \tilde{C} \geq \tilde{B} + \tilde{D}$

In this paper, we focus our use on Maleki, Campos, Yager's F1, and Yager RFs, Therefore, we limit the explanation to these functions only:

3.9.1. Maleki Ranking Function

The Maleki RF is a method used in fuzzy logic to rank fuzzy numbers. It was introduced by Maleki in a paper titled "A new ranking method for fuzzy numbers based on left and right areas."The Maleki RF is based on the idea of comparing the areas to the left and right of each fuzzy number's graph. The basic principle is that a fuzzy number with a larger area to its left and a smaller area to its right should be ranked higher than fuzzy numbers with different area distributions [6, 22, 23].
Therefore, let $\tilde{A} = (a^l, a^u, \alpha, \beta)$ be a TrFN, then the RF is:

$$R(\tilde{A}) = \int_0^1 (inf\tilde{A}_\lambda + sup\tilde{A}_\lambda) d\lambda \tag{18}$$

This reduces to:

$$R(\tilde{A}) = a^l + a^u + \frac{1}{2}(\beta - \alpha) \tag{19}$$

Remark: $R(\tilde{A}) = a^l + a^u$, where $\tilde{A}$ be symmetric TrFN [24, 25].

3.9.2. Campos Ranking Function

The Campos RF is another method used in fuzzy logic to rank fuzzy numbers. It was introduced by Campos et al. in their paper titled "A new ranking method for fuzzy numbers based on similarity measure."

Let $\tilde{A} = (a^l, a^u, \alpha, \beta)$ be a TrFN, then the ranging function is:

$$R(\tilde{A}) = a^l + \lambda \left[(a^u - a^l) + (\frac{(a^l - \alpha) + (a^u + \beta)}{3} \right] - \frac{(a^l - \alpha)}{3} \tag{20}$$

The parameter λ, which belongs to the interval [0,1], can be understood as a measure of optimism or pessimism that the decision-maker must choose. A value close to 1 suggests an optimistic perspective, while a value close to 0 implies a pessimistic viewpoint.

3.9.3. Yager's F1 Ranking Function

Yager's F1 RF is a method used to rank fuzzy numbers introduced by Ronald R. Yager, a prominent figure in the field of fuzzy logic and decision-making. The function is part of a family of RFs proposed by Yager.The F1 RF is designed to rank fuzzy numbers based on their centroid values. The centroid of a fuzzy number represents a measure of its "center of mass" or "average" value. The idea is that fuzzy numbers with higher centroid values are ranked higher than those with lower centroid values.

Let $\tilde{A} = (a^l, a^u, \alpha, \beta)$ represent a TrFN. The RF is specified as follows [26]:

$$R(\tilde{A}) = \frac{1}{3}\left(\frac{\left[(a^u)^2 - (a^l)^2\right] + [(a^u + \beta)^2 - (a^l - \alpha)^2] + [a^u \cdot (a^u + \beta) - a^l \cdot (a^l - \alpha)]}{(a^u - a^l) + \left((a^u + \beta) - (a^l - \alpha)\right)}\right) \quad (21)$$

3.9.4. Yager Ranking Function:

Let $\tilde{A} = (a^l, a^u, \alpha, \beta)$ represent a TrFN. The RF is specified as follows [26]:

$$R(\tilde{A}) = \frac{\left(\int_0^1\left(a^l - \alpha L^{-1}(\lambda)\right)d\lambda + \int_0^1\left(a^U + \beta R^{-1}(\lambda)\right)d\lambda\right)}{2} \quad (22)$$

This reduces to:

$$R(\tilde{A}) = \frac{\left(a^l + a^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)}{2} (23)$$

3.10. Fuzzy linear programming:

A standard formulation of crisp LP is expressed as:

$$\text{Maximize or minimize } z = \sum_{j=1}^{n} c_j x_j$$

Such that:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1,2,.,m \qquad x_j \geq 0 \quad (24)$$

where $c_j \in R^n, b_i \in R^m, \; and a_{ij} \in R^{n\times m}$

In the context of FLP, the conventional crisp parameters in the LP model are replaced with fuzzy numbers. This transforms the crisp LP model into a FLP model.The study concentrates on tackling FLP problems with inequality constraints, where the parameters $\tilde{c}_j$, $\tilde{b}_i$, and $\tilde{a}_{ij}$ are represented as fuzzy numbers. This scenario is deemed the most complex among other possibilities, as uncertainties are pervasive in each of these parameters. Following this, we proceed to tackle the FLP problem utilizing different RFs:

a- When applying the Maleki RF, the LP problem assumes a distinct form:

$$MaxorMinz = \sum_{j=1}^{n}[c_j{}^l + c_j{}^u + \frac{1}{2}(\beta - \alpha)] \; x_j$$

Such that:

$$\sum_{j=1}^{n}[a_{ij}{}^l + a_{ij}{}^u + \frac{1}{2}(\beta - \alpha)]x_j \leq \left[b_i{}^l + b_i{}^u + \frac{1}{2}(\beta - \alpha)\right], i = 1,2,\ldots,m, x_j \geq 0 \quad (25)$$

b- When applying the Campos RF, the LP problem assumes a distinct form:

$$MaxorMinz = \sum_{j=1}^{n} [c_j{}^l + \lambda \left[(c_j{}^u - c_j{}^l) + \frac{(c_j{}^l - \alpha) + (c_j{}^u + \beta)}{3} \right] - \frac{(c_j{}^l - \alpha)}{3}] \; x_j$$

Such that:

$$\sum_{j=1}^{n} [\, a_{ij}{}^l + \lambda \left[(a_{ij}{}^u - a_{ij}{}^l) + \frac{(a_{ij}{}^l - \alpha) + (a_{ij}{}^u + \beta)}{3} \right] - \frac{(a_{ij}{}^l - \alpha)}{3}] x_j \le \left[b_i{}^l + \lambda \left[(b_i{}^u - b_i{}^l) + \frac{(b_i{}^l - \alpha) + (b_i{}^u + \beta)}{3} \right] - \frac{(b_i{}^l - \alpha)}{3} \right] \; , i = 1,2,\ldots, m x_j \ge 0 \quad \text{and } \lambda \in [0,1] \qquad (26)$$

c- When employing Yager's F1 RF, the LP problem adopts a specific form:

$$MaxorMinz = \sum_{j=1}^{n} \frac{1}{3} \left[\left(\frac{\left[(c_j{}^u)^2 - (c_j{}^l)^2 \right] + \left[(c_j{}^u + \beta)^2 - (c_j{}^l - \alpha)^2 \right]}{(c_j{}^u - c_j{}^l) + \left((c_j{}^u + \beta) - (c_j{}^l - \alpha) \right)} \right) + \left(\frac{\left[c_j{}^u . (c_j{}^u + \beta) - c_j{}^l . (c_j{}^l - \alpha) \right]}{(c_j{}^u - c_j{}^l) + \left((c_j{}^u + \beta) - (c_j{}^l - \alpha) \right)} \right) \right] x_j$$

Such that:

$$\sum_{j=1}^{n} \frac{1}{3} \left[\left(\frac{\left[(a_{ij}{}^u)^2 - (a_{ij}{}^l)^2 \right] + \left[(a_{ij}{}^u + \beta)^2 - (a_{ij}{}^l - \alpha)^2 \right]}{(a_{ij}{}^u - a_{ij}{}^l) + \left((a_{ij}{}^u + \beta) - (a_{ij}{}^l - \alpha) \right)} \right) + \left(\frac{\left[a_{ij}{}^u . (a_{ij}{}^u + \beta) - a_{ij}{}^l . (a_{ij}{}^l - \alpha) \right]}{(a_{ij}{}^u - a_{ij}{}^l) + \left((a_{ij}{}^u + \beta) - (a_{ij}{}^l - \alpha) \right)} \right) \right] x_j \le \frac{1}{3} \left(\frac{\left[(b_i{}^u)^2 - (b_i{}^l)^2 \right] + \left[(b_i{}^u + \beta)^2 - (b_i{}^l - \alpha)^2 \right] + \left[b_i{}^u . (b_i{}^u + \beta) - b_i{}^l . (b_i{}^l - \alpha) \right]}{(b_i{}^u - b_i{}^l) + \left((b_i{}^u + \beta) - (b_i{}^l - \alpha) \right)} \right), \text{ i= 1,2,......,m.} (27) x_j \ge 0$$

d- When employing the Yager RF, the LP problem adopts a specific form:

$$MaxorMinz = \sum_{j=1}^{n} \frac{1}{2} \left(c_j{}^l + c_j{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta \right) x_j$$

Such that:

$$\sum_{j=1}^{n} \frac{1}{2} \left(a_{ij}{}^l + a_{ij}{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta \right) x_j \le \frac{1}{2} \left(b_i{}^l + b_i{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta \right) \quad , i = 1,2,\ldots, m x_j \ge 0 \qquad (28)$$

These functions are designed to offer a structured approach for systematically comparing and arranging fuzzy numbers, taking into account both their membership and nonmembership. In the context of LP with fuzzy parameters, RFs play a crucial role in converting fuzzy constraints or objectives into crisp ones, facilitating the application of traditional LP techniques.

After formulating these FLP problems, the next step typically involves applying appropriate solution methodologies or algorithms to solve them, considering the inherent uncertainties in the problem data. These solution methodologies might involve fuzzy optimization techniques, RFs, or other approaches specifically designed for handling FLP problems.

3.11. Algorithm for Solution FLP PROBLEM with Ranking Function

Within this section, a new approach is utilized to find the optimal solution for the specified type of FLP problem:

$$Max\ (or Min\ )\ (\tilde{C}^T \otimes X)$$

Such that:

$$\tilde{A} \otimes X \ \le\ \tilde{B}, \qquad (29)$$

X is a non-negative fuzzy value.

The steps outlined in the proposed approach are as follows:

- **Step 1:** Given the FLP problem with inequality constraints, where the $\tilde{c}_j$, $\tilde{b}_i$, and $\tilde{a}_{ij}$ are representedas fuzzy numbers, we substitute such values into the problem formulation: $\tilde{C}^T = [\tilde{C}_j]_{1\times n}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$ and $\tilde{B} = [\tilde{b}_i]_{m\times 1}$, as defined in Section (3.10).
- **Step 2:** Utilizing the RF as defined in Section (3.9) and the FLP framework outlined in Section (3.10), convert the problem into a crisp LP problem.
- **Step 3:** Subsequently, we determine the optimal solution of the obtained crisp LP problem using known methods.

4. Results

The manufacturing process involves three products, namely P1, P2, and P3, which undergo processing across four distinct machines: M1, M2, M3, and M4. The time intervals needed to produce a single unit of each product and the daily capabilities of the machines is as follows:

Table 1: Data of time requred of each product and daily capacity of the machines.

Time Needed per unit (in minutes)												
Machines	**P1**				**P2**				**P3**			
M1	12	16	2	3	13	16	2	1	12	16	3	5
M2	14	20	2	2	....				13	18	2	3
M3	12	17	3	2	15	20	2	3	....			
M4	15	18	2	3	14	17	2	1	16	18	2	2
Ptrofit/Rs	13	15	2	2	12	14	3	2	15	18	3	2

Table 2: Daily capacity of the machines.

Machine Capasity (min/day)				
	Capasity			
M1	490	510	9	8
M2	470	490	10	6
M3	480	505	7	8
M4	388	425	5	8

It is worth noting that daily time availability may vary due to factors like machine breakdowns or overtime work, while profit margins for each product can fluctuate based on price changes. However, the company aims to maintain profits near: Rs.14 for P1, Rs.13 for P2, and Rs.16 for P3.
The objective is to determine the optimal production quantities for each product per day, maximizing overall profit. All produced quantities are assumed to be sold in the market.
Given the uncertainty in both product profits and machine time availability, determining production quantities becomes uncertain. Thus, we choose to model the problem as a FLP problem, using TrFNs

to represent uncertain values. For instance, the profit for P1, approximately 14, is represented as [13, 15, 2, 2]. Similarly, other parameters are depicted as symmetric TrFNs, considering the problem's characteristics and any additional requirements. Consequently, we formulate the FLP problem as follows:

Let us define the decision variables:

x_1: the quantity of Product 1 (P1) to manufacture per day

x_2: the amount of Product 2 (P2) to produce daily

x_3: the volume of Product 3 (P3) to manufacture each day

The objective is to maximize profit, considering the variability in profit for each product. We will aim to keep the profit close to the target values:

$$Max z = (13,15,2,2)x_1 + (12,14,3,2)x_2 + (15,18,3,2)x_3$$

Subject to:

$$\begin{gathered}(12,16,2,3)x_1 + (13,16,2,1)x_2 + (12,16,3,4)x_3 \leq (490,510,9,8)\\ (14,20,2,2)x_1 + (13,18,2,3)x_3 \leq (470,490,10,6)\\ (12,17,3,2)x_1 + (15,20,2,3)x_2 \leq (480,505,7,8)\\ (13,15,2,2)x_1 + (12,14,3,2)x_2 + (15,18,3,2)x_3 \leq (388,425,5,8)\\ x_i \geq 0, \qquad for i = 1,2,3\end{gathered}$$

So, by using RFs to convert fuzzy numbers into crisp values, you can solve the resulting crisp LP problem to obtain an optimal solution that considers the uncertainties present in the original fuzzy problem.

a. Using the proposed method (Maleki RF), the aforementioned FLP problem is transformed into the following crisp LP:

$$Max z = 28x_1 + 25.5x_2 + 32.5x_3$$

Subject to:

$$\begin{gathered}28.5x_1 + 28.5x_2 + 28.5x_3 \leq 999.5\\ 34x_1 + 31.5x_3 \leq 958\\ 28.5x_1 + 35.5x_2 \leq 985.5\\ 33.5x_1 + 30.5x_2 + 34x_3 \leq 814.5\\ x_i \geq 0, \qquad for i = 1,2,3\end{gathered}$$

The optimal solution for the above LP is as follows: ($x_1 = 0$, $x_2 = 0$, $x_3 = 23.96$, and $z = 778.57$).

b. Using the proposed method (Campos RF), when $\lambda = 0.5$, the aforementioned FLP problem is transformed into the following crisp LP:

$$Max z = 17.83x_1 + 16.5x_2 + 18x_3$$

Subject to:

$$\begin{gathered}15.5x_1 + 15.5x_2 + 15.8x_3 \leq 506\\ 18.7x_1 + 17.2x_3 \leq 486\\ 16.2x_1 + 19.2x_2 \leq 499\\ 17.8x_1 + 16.5x_2 + 18x_3 \leq 415\\ x_i \geq 0, \qquad for i = 1,2,3\end{gathered}$$

The optimal solution for the above LP is as follows: ($x_1 = 23.31$, $x_2 = 0$, $x_3 = 0$, and $z = 415.7$). It is worth noting that, we can test all values of parameter λ, which belongs to the interval [0,1].

c. Using the proposed method (Yager's F1 RF), the aforementioned FLP problem is transformed into the following crisp LP:

$$Max z = 8.72727x_1 + 12.7037x_2 + 16.2121x_3$$

Subject to:

$$14.2821x_1 + 14.222x_2 + 14.2889x_3 \leq 499.725$$
$$17x_1 + 15.7778x_3 \leq 478.905$$
$$14.222x_1 + 17.7778x_2 \leq 492.769$$
$$16.7879x_1 + 15.222x_2 + 17x_3 \leq 407.287$$
$$x_i \geq 0, \qquad for i = 1,2,3$$

The optimal solution for the above LP is as follows: ($x_1 = 0$, $x_2 = 0$, $x_3 = 23.96$, and $z = 388.36$).

d. Through the application of the proposed method (Yager RF), the FLP problem mentioned above is converted into the following crisp LP:Then by using simplex method we get the optimal

$$Max z = 13.8667x_1 + 12.4667x_2 + 15.9667x_3$$

Subject to:

$$14.2x_1 + 14x_2 + 14.1x_3 \leq 499.1$$
$$16.9x_1 + 15.7x_3 \leq 478$$
$$14x_1 + 17.7x_2 \leq 492.4$$
$$16.7x_1 + 15x_2 + 16.9x_3 \leq 407.2$$
$$x_i \geq 0, \qquad for i = 1,2,3$$

Then, using the simplex method, the optimal solution for the above LP is as follows:
($x_1 = 0$, $x_2 = 0$, $x_3 = 24.09$, and $z = 384.79$).

5. Discussion

In LP with fuzzy parameters, RFs help in converting fuzzy constraints or objectives into crisp ones, which can then be handled using traditional LP techniques. Overall, RFs provide a structured approach to handling fuzzy numbers in LP and decision-making contexts, enabling the integration of uncertainty and imprecision into optimization and decision-making processes. Hence, the estimation of the maximum benefit in the context of solving a crisp LP problem converted from a FLP problem depends on the RF selected to convert fuzzy numbers into crisp values.

6. Conclusion

This paper presents a novel approach to tackling FLP problems, where coefficients of the objective function, right-hand side values, and coefficients of constraints are represented as fuzzy numbers. The proposed method involves ranking these fuzzy parameters using distinct RFs, including the Maleki, the Campos, Yager's F1, and the Yager linear RF. By ranking these fuzzy parameters, the inherent uncertainties and ambiguities associated with fuzzy numbers are effectively managed. The main goal of the proposed method is to determine an optimal solution to the FLP problem. To achieve this, the ranked fuzzy parameters are utilized to formulate crisp LP problems, which can then be solved using conventional LP techniques. By transforming the fuzzy problem into a series of crisp problems, the proposed method facilitates the application of well-established optimization algorithms to identify optimal solutions. Through a systematic approach to address the inherent fuzziness in problem parameters, the proposed method offers a robust framework for obtaining optimal solutions in scenarios characterized by uncertainties and vagueness.

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